5. Beams

Introduction:

Apart from axial and torsional forces there are other types of forces to which members may be subjected. In many instances in structural and machine design, members must resist forces applied laterally or transversely to their axes. Such members are called beams. The main members supporting floors of buildings are beams, just as an axle of a car is a beam. Many shafts of machinery act simultaneously as torsion members and as beams. With modern materials, the beam is a dominant member of construction. The determination of the system of internal forces necessary for equilibrium of any beam segment will be the main objective of this chapter. For the axially or torsionally loaded members previously considered, only one internal force was required at an arbitrary section to satisfy the conditions of equilibrium. However, even for a beam with all forces in the same plane, i.e., a planar beam problem, a system of three internal force components can develop at a section. These are the axial force, the shear, and the bending moment. Determining these quantities is the focus of this chapter. The chapter largely deals with single beams. Some discussion of related problems of planar frames resisting axial forces, shears, and bending moments is also given. Only statically determinate systems will be fully analyzed for these quantities. Special procedures to be developed in subsequent chapters are required for determining reactions in statically indeterminate problems for complete solutions. Extensions to members in three-dimensional systems, where there are six possible internal force components, will be introduced in later chapters as ceded and will rely on the reader's knowledge of statics. In such problems at a section of a member there can be: an axial force, two shear components, two bending moment components, and a torque.

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5.1 Bending of Beams

5.1.1 Introduction:

Beams:

Bars Subjected to transverse loads.

Planar and slender members.

Supports:

Identified by the resistance offered to forces.

(a) Rollers/Links:

Resists forces in a direction along the line of action (Figure 5.1(a)).

(b) Pins:

Resists forces in any direction of the plane (Figure 5.1(b)).

(c) Fixed Support:

Resists forces in any direction (Figure 5.1(c)).

Resists moments.

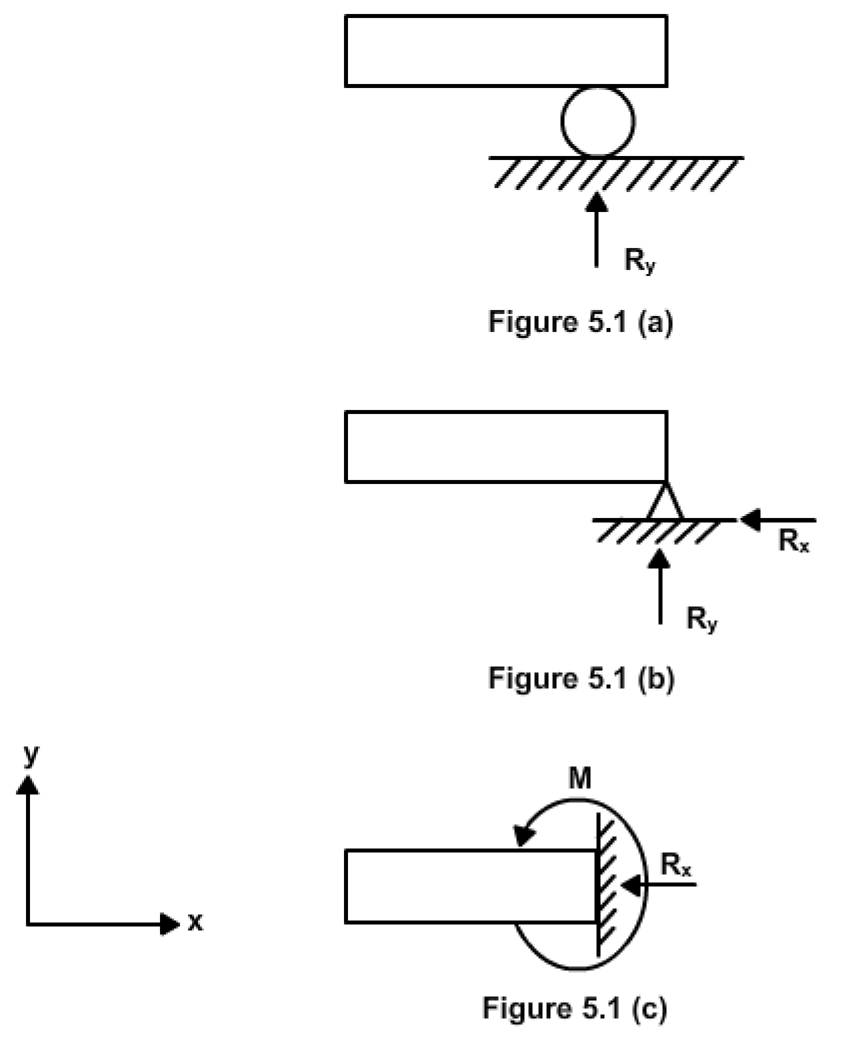

Figure 5.1 (a)

Figure 5.1 (b)

Figure 5.1 (c)

5.1.2 Classification of Beams

(a) Statically determinate or indeterminate.

Statically determinate - Equilibrium conditions sufficient to compute reactions.

Statically indeterminate - Deflections (Compatibility conditions) along with equilibrium equations should be used to find out reactions.

(b) Cross sectional Shapes - I,T,C or other cross sections.

(c) Depending on the supports used

1) Simply supported - pinned at one end and roller at the other (Figure 5.1.2(a))
2) Cantilever - fixed at one end and the other end free (Figure 5.1.2(b)).
3) Fixed beam - fixed at both ends (Figure 5.1.2(c)).

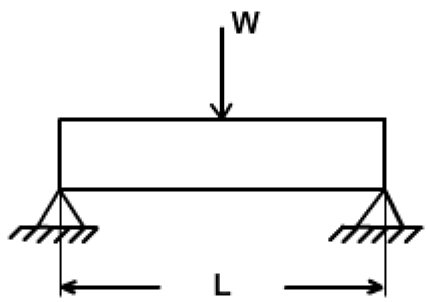

Figure 5.1.2(a)

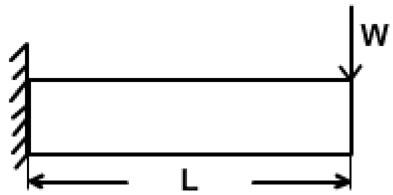

Figure 5.1.2(b)

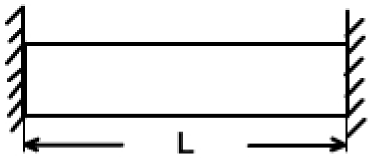

Figure 5.1.2(c)

Where W – loading acting, L – span.

5.1.3 Calculation of beam reactions

When all the forces are applied in a single plane, the three equations of static equilibrium are available for analysis.

$$\sum F_x = 0, \ \sum F_y = 0 \text{ and } \sum M_3 = 0$$

Employing these, the reactions at supports could be found out.

5.1.4 Procedures for Computing forces and moments

For a beam with all forces in one plane, three force components are internally developed.

- Axial force
- Shear
- Bending Moment

Procedures are to be established for finding these quantities.

Direct Method/Method of sections:

This method is illustrated by the following example

Find reactions at supports for the cantilever shown in figure 5.1.3(a) subjected to uniformly distributed load.

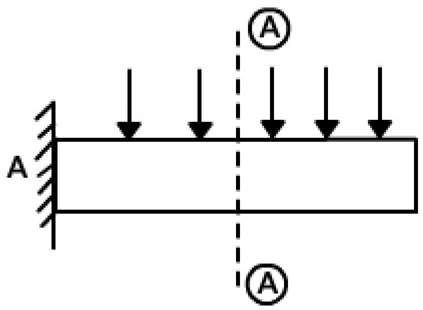

Figure 5.1.3(a)

Cut the cantilever along section A-A and obtain free body diagram as given in figure 5.1.3(b).

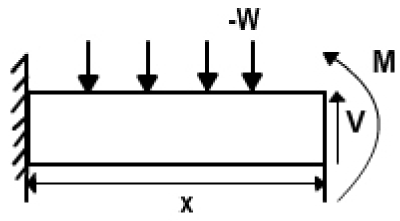

Figure 5.1.3(b)

The segment of the beam shown in figure 5.1.3(b) is in equilibrium under the action of external forces and internal forces and moments.

$$\sum F_y = 0$$

gives

$$V = -wx$$

$$\sum M = 0$$

$$\text{gives, } M\text{-}\frac{wx^2}{2} = 0$$

$$M = \frac{wx^2}{2}$$

Sign convention: Shear force

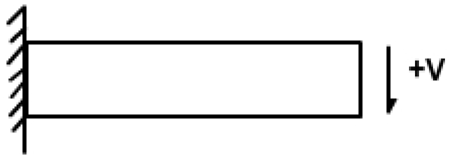

Figure 5.1.4(a)

Figure 5.1.4(b)

Figure 5.1.4(c)

Sign convention: Moments

Figure 5.1.5(a)

Figure 5.1.5(b)

As Shown in Figure 5.1.5(a), sagging (beam retains water) moment is positive, other wise bending moment is negative (Figure 5.1.5(b)).

5.1.5 Shear force and Bending Moment Diagrams (SFD & BMD)

Plot of shear and bending moment values on separate diagrams could be obtained.

Magnitude and location of different quantities can be easily visualized.

SFD & BMD are essential for designers to make decisions on the shape and size of a beam.

The worked out examples illustrate the procedure for plotting SFD and BMD by direct approach.

5.1.6 Shear force and BM Diagrams / (Alternate approach)

Beams Element: Differential equations of Equilibrium

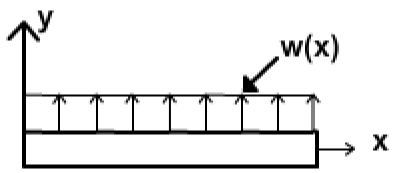

Figure 5.1.6

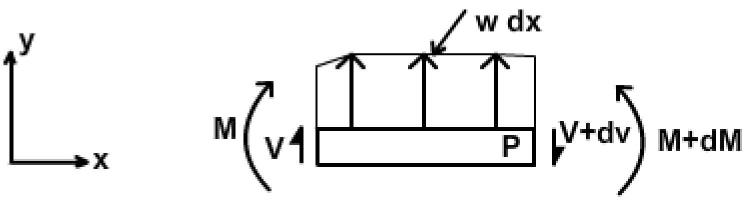

Figure 5.1.7

Free body diagram of element of length dx is shown Figure 5.17, which is cut from a loaded beam (Figure 5.1.6).

$\sum F_y = 0$ gives

$V + wdx - (V + dv) = 0$

$$\text{ie } \frac{dv}{dx} = w \qquad 5.1.1$$

$\sum M_p = 0$ gives

$M + dM - Vdx - M - (wdx).(dx/2) = 0$

$$\text{ie}\ \frac{dM}{dx} = V \qquad 5.1.2$$

Substituting equations 5.1.2 in 5.1.1

$$\frac{d^2M}{dx^2} = V \qquad 5.1.3$$

Integrating 5.1.2

$$V = \int_0^x w dx + c_1$$

Integrating 2

$$M = \int_0^x V dx + C_2 + M_e$$

M_e - External moment acting.

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5.2 Theory of Bending

5.2.1 General theory

Plane sections normal to the axis before bending remain plane and normal after bending also, as shown in Figure 5.2.1.

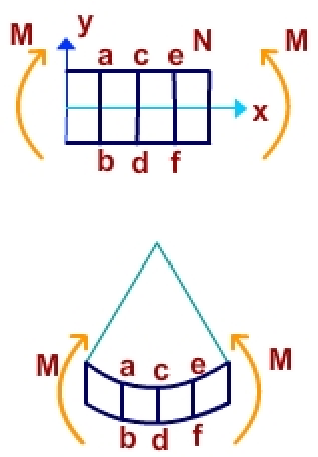

Figure 5.2.1

From Figure 5.2.1, ab, cd efs are sections which remain plane and normal. Beam is subjected to pure bending (no shear). Longitudinal top fibers are in compression and bottom fibres in tension.

Layer of fibres in between which is neither in tension or compression, is called the neutral surface. Neutral axis is the intersection of such a surface with the right section through the beam.

Assumptions of the theory of bending

Deflection of the beam axis is small compared to span of the beam.

Shear strains, along the plane xy are negligible.

Effect of shear stress in the plane $xy\left(\tau_{xy}\right)$ on normal stress $\left(\sigma_x\right)$ is neglected.

Note: Even through pure bending is assumed, distribution of normal stresses at any given cross section does not get significantly changed due to non uniform bending.

For pure bending of a beam, beam axis deforms into part of a circle of radius ρ; for an element defined by an infinitesimal angle d , the fiber length is given by (refer figure 5.2.2)).

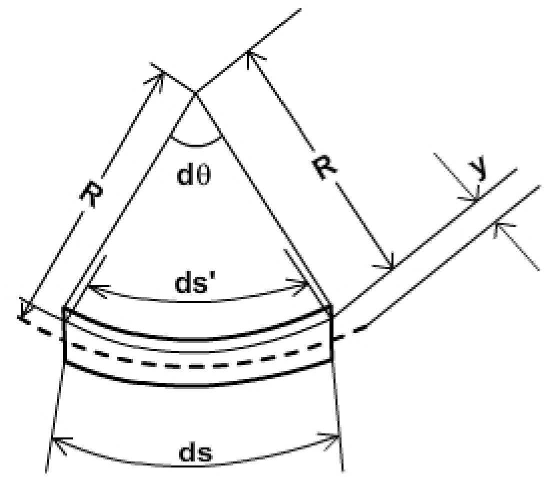

Figure 5.2.2

ds = R dθ

$\frac{d\theta}{ds} = \frac{1}{R} = k$, where

R - Radius of Curvature

k - Axis Curvature

For a fiber located at radius R' = R – y

$ds' = (R - y) d\theta$

Strain, $\varepsilon_x = \frac{ds - ds'}{ds}$

$\varepsilon_x = -ky$

5.2.2 Elastic Flexure Formula

By Hooke's Law,

$\sigma_x = E\varepsilon_x = -Eky$

$\sum F_x = 0$ gives

$$-Ek\int ydA = 0$$

i.e. neutral axis possess through the centroid of the cross section (Ref Figure 5.2.3.)

$\sum M = 0$ gives

$$M_z = E_k \int y^2.dA$$

$$= Ek\ I_z$$

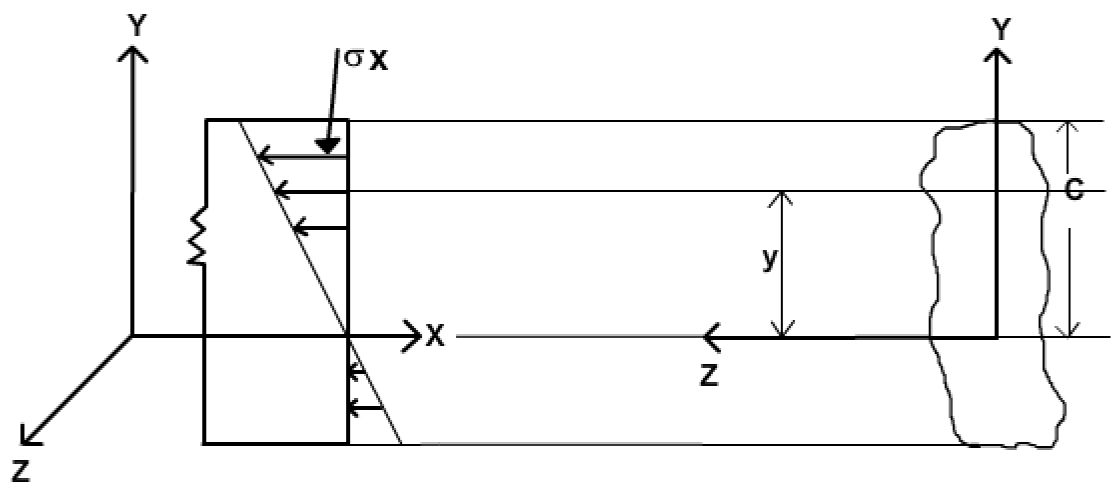

Figure 5.2.3

This gives

$$k = \frac{M}{E\ I_z}$$

$$\sigma_x = \frac{-M_z y}{I_z}$$

$$\sigma_{max} = \frac{M_z C}{I_z}$$

5.2.3 Beams of Composite Cross section

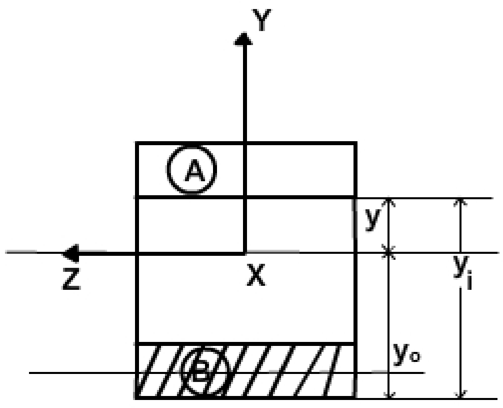

Figure 5.2.4

For beams of composite cross section $\sigma_x = -E_i ky$ for the i^th material in the composite.

$y = y_i - y_0$

y_0 is the location of neutral axis from the bottom of the beam.

y_i is the location of neutral axis of the i^th material. In the figure $y_i = y_A$, from this, we get

$$y_0 = \frac{\int_A E_i \ y_i \ dA}{\int E_i \ dA}$$

Where A the area of cross section of the corresponding material. The procedure for analyzing beams of composite cross section is illustrated in worked out examples.

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